Finite difference computation of the Elliptic Poisson Partial Differential Equation

# Purpose

## Physical context

The purpose is to solve the Elliptic Poisson Partial Differential Equation with help of the Finite Difference method. More particularly we solve it with a five point difference scheme in two dimensional space and Dirichlet Boundary Condition. The explanation about the method has been taken from [[1](#Bur10)] and [[2](#Cha10)].

The problem comes historically from the following Heat Equation of Fourier:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

In two dimensional spaces and for any general real function in may be written as below:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

We note that the function is not anymore dependent of time. For instance, concerning the Heat Equation, it corresponds to the steady-state regime. In the particular case of, the equation (2) is known as the Laplace Equation.

## Discretization

The problem is discretized over a rectangular area with the abscissa: and. The step over the abscissa is given by and over ordinates by. The coordinate of any point in the grid is therefore given by:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

The Taylor series on let us generate the symmetrical difference formula for the second order partial derivation of.

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

And the Poisson Equation may be written as:

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

## The finite difference method

### Finite difference scheme

By considering the higher order terms as negligible and by multiplying each side by, the equation (5) lead to the following general scheme in the case of an irregular step-size grid:

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

This scheme has ***truncation error***:

In the particular case of a regular grid of identical step size along the abscissa and the ordinates one obtain the now widespread formula:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

# Protocol

# Result

# Bibliography

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| [1] | Burden R L. and Faire J D, *Numerical Analysis*, 9th ed.: Brooks/Cole, 2010. |
| [2] | S C Chapra and R P Canale, *Numerical Methods For Engineers*, Sixth Edition ed., Mac Graw Hill, Ed., 2010. |

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