Finite difference computation of the Elliptic Poisson Partial Differential Equation

# Purpose

The purpose is to solve the Elliptic Poisson Partial Differential Equation with help of the Finite Difference method. More particularly we solve it with a five point difference scheme in two dimensional space and Dirichlet Boundary Condition.

The problem comes historically from the following Heat Equation of Fourier:

In two dimensional spaces and for any general real function in may be written as below:

We note that the function is not anymore dependent of time. For instance, concerning the Heat Equation, it corresponds to the steady-state regime. If , the equation is known as Laplace Equation.

# Protocol

# Result

## The basic Taylor formula

We will now give the basic formula of a Taylor expansion and other more convenient forms of these formulas that once assemble allow producing schemes.

For our function of a scale variable the Taylor formula sufficiently near a point may be written the following:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

This may be rewritten the following by letting:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

## Different variation on the extension

It is possible to derive a wide range of Taylor expansion depending on the point we wish to calculate it. The only thing to care of is that the variable tends to a when h tends to 0.

From (2) we can derive:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

These expansions will serve as basis bricks to build computation schemes.

## Remark

1. These estimations depend on
2. These estimations are local. The more h is small the more they will be precise.
3. The function is the truncation error. It will give an idea of how much decimal are significant in the result. Here also if h is sufficiently small, it will increase the degree of precision of the calculation (a trade-off exists between the size of h, the memory handled and the computation time).

# Bibliography

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| [1] | LeVeque R J., *Finite Difference Methods for Ordinary and Partial Differential Equations*. Philadelphia: Society for Industrial and Applied Mathematics (SIAM), 2007. |

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